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A Method for Estimation of ASFR for the Intercensal Period from the CEB Data

Introduction

ESTIMATION of vital rates in most of the developing countries is handicapped due to lack of complete and reliable vital statistics. Though the registration of births and deaths in some of these countries started as early as the beginning of the nineteenth century, it is still far from satisfactory in respect of its completeness and reliability. As such one has to take recourse to indirect methods to obtain the estimates of vital rates by using age distribution of the population obtained through censuses or surveys and some times using retrospective questions on fertility and mortality.

Recently some methods have been developed notably by Bogue (1971) and Arretx (1973) to estimate ASFR during the intercensal period by using the responses on children ever born (CEB) question from two successive census. However, the methods presented by both Bogue and Arretx are conceptually extension of Mortara (1949). Bogue's estimates may be biased due to assumption of linear growth in CEB rates during the intercensal period because the assumption of linearity is likely to be violated due to the use of contraception, especially by women with higher parities. Arretx procedure again assumes a smooth increase in CEB rates of women belonging an age cohort and uses Brass Polynomial fertility curve to ascertain the age specific fertility rates by conventional age group. In this paper we apply another mathematical curve to describe the pattern of age specific fertility rates instead of the Brass Polynomial to derive the estimates of ASFR for the intercensal period on the lines of Arretx. The rationale for selecting this curve has been given by Ram (1980). The method is tested by applying the method to the data from U.S. Censuses from 1920-1950 and wherever possible they have been compared with the observed or the estimated values obtained by using Bogue's procedure.

Basic Concept with Alternative Mathematical Mode!

The procedure developed by Arretx is conceptually based on the principle advanced by Mortara (1949), which uses the information on CEB tabulated by age of the women from single census to estimate annual age specific fertility rates. The underlying assumption is that fertility has not changed in the past. Arretx (1973) proposes the use of two successive censuses to relax the assumption made by Mortara and assumes that the an age specific fertility follows one parameter mathematical function proposed by Brass (1968); namely

$$F(x) = C(x - a)(a + 33 - x)^2. \quad (1)$$

But in the present context where fertility is declining faster due to the Government supported national family planning programmes the above curve may not represent the variation in age pattern of fertility to suit the changed phenomenon, following function is suggested:

$$f'(x) = K(x - a)(b - x)e^{-\lambda(x-a)}. \quad (2)$$

where K is the scale parameter and A is the shape parameter; a and b are the lower and upper limits of the reproductive life of the woman which are generally taken as 15 and 50, respectively. However, the general procedure of estimating these parameters is given in Ram (1980). Our purpose here is to use the above curve to estimate the age specific fertility rates from the CEB data on the lines of Arretx for two extreme values of A, namely A = .025 and A = .10. These extreme values are suggested from the empirical studies made by Ram (1980). The procedure of estimating ASFR is briefly as follows.

The average number of children born per woman in the age group $(x, x + 5)$, $({}_5h_x)$ corresponds to the cumulative fertility of the woman of age I in this age group. That is,

$$ {}_5h_x = H(I); \; x \leqslant I \leqslant x + 5. \quad (3)$$

Let us assume that one have cumulative fertility of the woman at any time for central age of each *quinquennial* age group. There the difference.

$$H^{z+10}(x + 12.5) - H^z(x + 2.5) = {}_{10}F_{x+2\cdot5} \quad (4)$$

denotes the number of births per woman in the period Z and $Z + 10$ to the cohort of women aged $x + 2.5$ at Z. Hence the annual intercensal fertility rates are given by

$$ {}_{10}f_{x+2.5} = 1/10 \; X \; {}_{10}F_{x+2.5}. \tag{5} $$

Also since $f'(x)$ denotes the fertility at age x,

$$ F(x) = \int_{15}^{x} f'(x)dx $$

represents the number of children per woman at the exact age x. Further if we define

$$ G(x) = \int_{15}^{x} F(x)dx \tag{6} $$

then $\frac{1}{5}[G(x + 5) - G(x)] = {}_5h_x$ is approximately equal to $F(E)$ where E is an age between ages x and $x + 5$. If it is assumed that age structure in the census is comparable to that in the model, then within each *quinquennial* age group

$$ {}_5h_x = H(I) = F(E). $$

By using again the mathematical function it is possible to convert the value of $F(E)$ into a value of $F(x + 2.5)$, Arretx calls this factor as J_i for $i = 1, 2. . .7,$ standard *quinquennial* age groups. By these J_i factors it is possible to transform the known accumulated fertility for women classified by five year age groups into the accumulated fertility at exact ages (the middle age).

Further theoretically it can be shown that

$$ {}_{10}f_{x+2.5} = \tfrac{1}{10}[F(x + 12.5) - F(x + 2.5)] \tag{7} $$

and also

$$ {}_5f_x = \tfrac{1}{5}[F(x + 5) - F(x)]. \tag{8} $$

Now it may be easily shown that (if fertility does not change much)

$$ \tfrac{1}{10}[F(27.5) - F(17.5)] = \tfrac{1}{5}[F(25) - F(20)] = {}_5f_{20} $$
$$ \tfrac{1}{10}[F(47.5) - F(37.5)] = \tfrac{1}{5}[F(45) - F(40)] = {}_5f_{40}. $$

Hence it is possible to derive *Ki* factor by the relation

$$K_i = \frac{\frac{1}{5}\,[F(x+10) - F(x+5)]}{\frac{1}{10}[F(x+12.5) - F(x+2.5)]} = \frac{_5f_{x+5}}{_{10}f_{x+2\cdot5}} \tag{9}$$

for i = 2, 3, 4, 5, 6.

For the first age group and the last age group we obtain

$$K_1 = \frac{_{2\cdot5}f_{17\cdot5}}{_{7\cdot5}f_{15}} \tag{10}$$

and

$$K_7 = \frac{_{2\cdot5}f_{45}}{_{7\cdot5}f_{42\cdot5}}\,. \tag{11}$$

The values of E_r an age for which average children born per woman in a given' age group equals the cumulative fertility are computed by using the mathematical function of Arretx and the expression (2) for two extreme values of X. The values are given in Table 1.

TABLE 1—VALUES OF *E* WHICH SATISFY THE RELATION *F(E)* = 5^{h}x

Quinquennial age group	*Value of E determined by our mathematical function when is epual to*		*Brass polynomial**
	.025	.100	
15-20	17.82	17.75	17.8
20-25	22.57	22.50	22.6
25-30	27.51	27.44	27.5
30-35	32.48	32.41	32.4
35-40	37.44	37.38	37.4
40-45	42.39	42.32	42.3
45-50	47.14	47.07	46.9

*Arretx C. (1973).

It is clear from the Table 1 that value of E is independent of A. Tn fact in computation of the exact age we consider the distribution of births within each *quinquennial* age group which may not be affected by minor curvature of the curve which our model suggests due to exponential function in X. It is also observed that there is no significant change in values of £ with respect to value of a, the initial age of reproduction again the factor *Ji* are calculated for different values of A but only their average value for each age group are being given in Table 2 which also indicates that $J_i's$ are not function of the shape parameter particularly for ages 30 and above. Since there is no significant difference between the value of E and the central age and $J_t's$ are the ratio in accumulated fertility at these ages, they do not reflect the changes in age pattern of fertility.

TABLE 2—FACTORS J_i TO CONVERT THE ACCUMULATED FERTILITY BY QUINQUENNIAL AGE GROUPS **INTO** THE CORRESPONDING FERTILITY FOR THE CENTRAL EXACT AGE

Age	*Value of Ji from our model with equal to*		*Value of J_t for average value*	*Standard deviation $J_i's$*	*From Brass* polynomial s = 15*
	X = .025	X = .100			
15-20	.79678	.84222	.81978	.01396	.8068
20-25	.98502	1.00023	.99306	-00467	.9781
25-30	.99881	1,00483	1.00220	.00185	1.0000
30-35	1.00165	1 .00373	1.00297	.OOC64	1.0063
35-40	1.00245	1.00245	1.00261	.000087	1.0033
40-45	1.C0272	1.00149	1.00212	.00038	1.0026
45-50	1.00300	1.00090	1.00180	.00065	1.0012

*Arretx 1973.

Now the factors *Ki* which are used to convert intercensal fertility rates into annual fertility rates for 5 year age groups, have been calculated for different values . But again these are given only for two extreme values of A,; and an average value in Table 3.

The value of standard deviation (S.D.) given in Table 3 clearly give the effect of shape parameter. It is, however, clear that the effect of $K_i'S$ is more towards both the ends of the reproductive period. In practice it has been observed

TABLE 3—FACTOR K_i USED TO CONVERT INTERCENSAL RATES INTO ANNUAL FERTILITY RATES FOR 5 YEAR AGE GROUPS

Age	Value of K_i for $\lambda = .025$	Value of K_i for $\lambda = .100$	Average value of K_t	Standard deviation	Brass method*
15-20	1.06299	1,13277	1.09892	.02147	1.0720
20-25	1.04641	1.06984	1.06067	.00734	1.05648
25-30	1.02689	1.01583	1.02404	.00369	1.03351
30-35	1.01904	.99073	1.00744	.00882	1.02090
35-40	1.01513	.97086	.99532	.0137	1.00458
40-45	1.01359	,94353	.98025	.02158	.95281
45-50	1.00999	.92402	.96749	.02643	.82058

*Same as Table 2.

that the tempo of fertility (either in positive direction or negative direction) is high at both end of the reproductive period which might be taken into account by *Ki*. But it does not happen so in case of Brass Polynomial.

Application

The data on children ever born (CEB) per woman by age has been taken from the population censuses of United States for 1920, 1930, 1940, 1950 and 1960. In general, rates of CEB based on 1960 census have a high degree of comparability with similar data from previous censuses (U.S. Census of Population, 1960 women by number of CEB page X). Despite the various factors expected to cause some differences, such as, variations in wording of the questions, processing of data, immigration and differential mortality., there is consistency in the rates for 1940, 50 and 60. This probably shows that variations in wording of questions had little effect on quality of data. In the 1950 census, 9.0 per cent of the women had no report on CEB (this figures refers to women ever married 15-59 years old) whereas corresponding figure for 1940 was 12.6 per cent. Under-reporting of the CEB shown above may be due to recall lapse by the women age 40 years and above. Another source of error which may affect our estimates is migration but no adjustment has been made because it has not deteriorated CEB significantly.

Inspite of various expected problems such as quality of data, international migration which might affect the estimates, the mathematical function described in equation (2) has been used to estimate the intercensal fertility rates for period 1920-30, 30-40, 40-50 and 1950-60. The estimates so obtained are shown in Table 4. It indicates the peculiar trend in tola! fertility rate (TFR) which otherwise too has been observed in 4.5 in the recent past. Age pattern of fertility estimated by using equation (2) and Brass Polynomial (1968) with observed age specific fertility rates for 1945 and 1955 are given in Table 4.

TABLE 4—ESTIMATED ASFR FOR THE INTERCENSAL PERIODS OF 1920-30, 1930-40, 1940-50 AND 1950-60 AND OBSERVED ASFR FOR 1945 AND 1955, UNITED STATES OF AMERICA

Age Group	*Estimates of ASFR for Periods from our- method*				*Bogue's+ Estimate for*	*Observed ASFR*	
	1920-30	*1930 40*	*1940-50*	*1950-60*	*1950-60*	*1945*	*7955*
15-20	.0479	.0414	.0596	.0914	.0483	.0511	.0903
20-25	.1378	.1121	.1480	.2004	.2235	.1389	.2416
25-30	.1493	,1197	.1487	.1794	.2117	.1322	.1902
30-35	.1142	.0888	.1027	.1073	.1322	.1002	.1160
35-40	.1080	.0630	.0563	.0620	.0634"	,0569	.0586
40-45	.0426	.0333	.0213	.0380	.02380	.0166	.0161
45-50	.0310	—	—	--	—	.0016	.0010
TFR	3.1544	2.2915	2.6830	3.3925	3.5145	2.4910	3.5740

\+ **Bogue, Donald, J.,** ***Demographic*** *Techniques of Fertility Analysis,* p. 64.
SOURCE: Vital Statistics of U.S., 1975, Vol 1, Natality, Table 6.

The significant difference between observed and estimated intercensal fertility rates can be seen in age group 20-25. However, the fertility rates during 1940-50 was over estimated in this age group whereas it was underestimated for the period 1950-60.

Concluding Remarks

The intercensal fertility estimates obtained through either using Brass Polynomial or equation (2) logically can not be compared with the age specific fertility rates for middle most period particularly in case when fertility is declining or increasing rapidly. It is observed for the U.S.A. (U.S., 1975) that the fertility increase during 1940-50 was 26 per cent whereas corresponding figure for 1950-60 was only 15 per cent. However, 5 years break up shows maximum increase during 1945-50 (19.4 per cent) followed by increase during 1950-55 (13.4 per cent). In such cases, it may be more meaningful if we had the average value of fertility rates for intercensal period. One reason for underestimation of fertility during 1950-60 may be illegitimate births which are relatively more in birth registration data than the census data (U.S., 1960).

However, the figures reflect no significant difference between the estimates obtained either through Brass polynomial or through the mathematical function. The total fertility rates for 1945 and 1955 are under or over estimated to the same extent by both the methods. But it is generally observed that the estimates at extreme ages derived by using function (2) are higher than those of Brass Polynomial ' It may be because Of the high tempo of fertility at both sides of the peak which would be taken into account by equation (2) but not by Brass Polynomial. Second main point which would narrow down the differences may be intercensal period during which there may not be substantial change m pattern and is subject to relative form at both points of time. It may certainly make difference when used in case when cumulative performance since beginning of reproduction is taken into the calculation viz. P/F ratio method (Brass *et al* 1968) That is, when the cohort concept is used for the analysis of cross sectional data. However, the results obtained indicate that the proposed curve can be used to the represent the pattern of fertility.

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